Journal of Engineering Physics and Thermophysics, Vol. 75, No. 1, 2002

COOLING OF A PLANE-PARALLEL DISPERSE LAYER

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UDC 536.33

In a continuation of previous works, in which the kinetics of heating of a plane-parallel light-diffusing layer was studied by numerical solution of differential equations of radiation transfer and heat conduction, we consider the process of cooling of an initially uniformly heated layer. Examples are presented for different optical and thermophysical characteristics. We suggest the concepts of the characteristic times determining which one can estimate a number of specific parameters of the processes of cooling of a medium without resorting to solution of differential equations. We have obtained a simple analytical solution of the problem for the case where the layer remains uniformly heated during the whole process of cooling. It is shown that under certain conditions the ratio of the volume concentration of particles in the medium to their diameter can be calculated from the rate of cooling.

In a series of works [1–5], we considered different problems of the kinetics of heating of light-diffusing and absorbing layers with account for the mechanisms of radiative heat transfer and heat conduction. This is the process of heating of the layer by external heat sources and by grids located inside the layer for different optical, thermophysical, and geometric characteristics, in a one-component medium or a multicomponent medium, and in the presence and absence of the spectral dependence of optical parameters. In this case, the temperature and radiation characteristics and the times of reaching the stationary mode were analyzed. Meanwhile, knowledge of the character of cooling of such a medium is of interest for many practical problems. In the present work, we study the problem indicated.

Problem Formulation and Assumptions. We consider a horizontal plane-parallel layer with a thickness x_0. If the medium is macrohomogeneous, then, in cooling, all points of its volume element have the same temperature. In the case of a multicomponent medium with dramatically different optical and, thermophysical characteristics of its components, the components will cool according to their own laws, and, consequently, they will have different temperatures. We restrict ourselves to consideration of the medium as a "gray" body, all optical characteristics of which are independent of the wavelength of light, lest the problem be complicated by taking account of the spectral characteristics of radiation. Let the medium itself consist of two components: a binding mass and equal particles in it. Propagation of radiation in a turbid medium in the two-flow approximation is described by the equation of transfer [6, 7]

$$\frac{dE_{\downarrow}}{dx} = 2\sigma\,(k_1 T_1^4 + k_2 T_2^4) - 2kE_{\downarrow} + 2\,(E_{\uparrow} - E_{\downarrow})\,\varphi_2 s_2\,, \tag{1}$$

$$\frac{dE_{\uparrow}}{dx} = -\,2\sigma\,(k_1 T_1^4 + k_2 T_2^4) + 2kE_{\uparrow} + 2\,(E_{\uparrow} - E_{\downarrow})\,\varphi_2 s_2\,. \tag{2}$$

Here $E_{\downarrow}$ and $E_{\uparrow}$ are the luminosities of the layer at the top and at the bottom at the depth x at the instant of time t, $\sigma = 5.67\cdot10^{-8}$ W·m^{-2}·K^{-4} is the Stefan–Boltzmann constant, k_1 is the absorption coefficient

B. I. Stepanov Institute of Physics, National Academy of Sciences of Belarus, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 75, No. 1, pp. 122–127, January–February 2002. Original article submitted May 14, 2001.

1062-0125/02/7501-0163$27.00

of the binding medium, k_2 and s_2 are the coefficients of absorption and scattering of the volume element which are caused by the particles, $k = k_1 + k_2$ is the resultant absorption coefficient, $\varphi_2 = (3 - x_1)/8$ is the portion of radiation backward scattered by the particles in the volume element which is subjected to diffuse radiation on one side, x_1 is the first coefficient in expansion of the scattering indicatrix of the volume element in Legendre polynomials, and T_1 and T_2 are the temperatures of the binding mass and of the particles in it.

We write the equations of heat transfer under the following assumptions: temperature is equalized instantly inside particles; particles are not in contact with each other and interact only with the binding medium. Under these conditions, the heat-conduction equation for the binding medium is

$$c_1 \rho_1 \frac{\partial T_1}{\partial t} = k_1 (E_\downarrow + E_\uparrow - 2\sigma T_1^4) + \eta_1 \frac{\partial^2 T_1}{\partial x^2} + n S_p \alpha_2 \eta_1 (T_2 - T_1) , \quad (3)$$

and for the light-diffusing component it is

$$v_2 c_2 \rho_2 \frac{\partial T_2}{\partial t} = k_2 (E_\downarrow + E_\uparrow - 2\sigma T_2^4) - n S_p \alpha_2 \eta_1 (T_2 - T_1) . \quad (4)$$

Here η_1, c_1, and ρ_1 are the coefficient of thermal conductivity of the binding medium and its heat capacity and density, respectively; similar characteristics with the subscript 2 refer to the light-diffusing component. Moreover, v_2 is the volume concentration of the particles, n is the number of particles in the unit volume, S_p is the surface area of one particle, and α_2 is the coefficient of heat exchange between a particle and the binding medium.

The boundary conditions for radiation flows near the upper and lower boundaries of the layer have the form

$$E_\downarrow (0, t) = E_\uparrow (x_0, t) = 0 , \quad (5)$$

and for the temperature of the binding medium near the upper and lower surfaces of the layer they are

$$\left. \frac{dT_1}{dx} \right|_{0, x_0} = \alpha_1 (T_1 - T_\infty) , \quad (6)$$

where T_∞ is the temperature of the environment from above and from below (constant with time) and α_1 is the coefficient of heat exchange between the binding medium and the environment.

The temperature which is constant within the entire medium and in its components is the initial condition

$$T_1 (x, t = 0) = T_2 (x, t = 0) = T_{in} , \quad (7)$$

where T_{in} is the initial temperature. For the particles to correspond to the condition of "grayness," we consider spherical particles with a diameter d much larger than the fundamental wavelengths of radiation in the medium. Under these conditions, their attenuation cross section is $\varepsilon_p = 2\pi d^2/4$, their scattering cross section is $s_p = (1 + \gamma)\pi d^2/4$, and their absorption cross section is $k_p = (1 - \gamma)\pi d^2/4$, where γ is the coefficient dependent on the complex refraction index of a particle. If the imaginary part of the complex refraction index is rather large, then the light, which has entered the particle, does not come out of it. When the real part of the refraction index is small, the scattering will be determined only by the diffraction component and $\gamma = 0$. Then $s_p = k_p = \pi d^2/4$ and $\varepsilon_p = \pi d^2/2$ and $s_2 = k_2 = n\pi d^2/4 = 3v_2/2d$ and $\varepsilon_2 = n\varepsilon_p = 3v_2/d$, where n is the

number of particles in unit volume. Thus, the optical properties of the considered particles remain constant within a wide range of wavelengths.

With account of the above, Eqs. (1)–(4) take on the form

$$\frac{dE_{\downarrow}}{dx} = 2\sigma k_1 T_1^4 + 3\sigma v_2 T_2^4/d - 2kE_{\downarrow} + 3v_2\,\varphi_2\,(E_{\uparrow} - E_{\downarrow})/d\,, \tag{8}$$

$$\frac{dE_{\uparrow}}{dx} = -\,2\sigma k_1 T_1^4 - 3\sigma v_2 T_2^4/d + 2kE_{\uparrow} + 3v_2\,\varphi_2\,(E_{\uparrow} - E_{\downarrow})/d\,, \tag{9}$$

$$c_1\,\rho_1\,\frac{\partial T_1}{\partial t} = k_1\,(E_{\downarrow} + E_{\uparrow} - 2\sigma T_1^4) + \eta_1\,\frac{\partial^2 T_1}{\partial x^2} + 6v_2\alpha_2\eta_1\,(T_2 - T_1)/d\,, \tag{10}$$

$$c_2\,\rho_2\,\frac{\partial T_2}{\partial t} = 3\,(E_{\downarrow} + E_{\uparrow} - 2\sigma T_2^4)/2d + 6\alpha_2\eta_1\,(T_2 - T_1)/d\,. \tag{11}$$

Solution of the problem is reduced to a numerical analysis of the system of differential equations of radiation transfer (8) and (9) and heat condition (10) and (11) and is performed on a personal computer according to a specially developed program. This program is based on the finite-difference approximation of derivatives with respect to space, which reduces the problem to a system of ordinary differential equations with respect to time for the temperature at each point of the considered layer [8, 9].

Analysis of the Results. The calculation results given below are obtained under the following conditions: $T_{\text{in}} = 1000$ K, $T_{\infty} = 300$ K, and $k_1 = 10$ m^{-1}. Since the particles have $d \gg \lambda$, we set φ_2 independent of the light wavelength λ and equal to 0.05. This value is quite realistic for large "soft" particles. For a wide range of materials $c\rho$ has a value of about 10^6 J·m^{-3}·K^{-1}. Therefore, we set $c\rho = c_1\rho_1 = c_2\rho_2 = 10^6$ J·m^{-3}·K^{-1}. From (10) and (11) it also follows that the kinetics of cooling depends on $t/c\rho$ rather than on t; in this connection, if the real value of $c\rho$ differs from the value indicated, then the correction for the time parameters and regularities which will be considered below is equal to $c\rho/10^6$. We can also normalize the real temperatures to T_{∞}, the densities of the light fields $E_{\downarrow} + E_{\uparrow}$ to $2\sigma T_{\infty}^4$, and the coordinates of the layer x to x_0 and use the characteristics $\overline{T}_1 = T_1/T_{\infty}$, $\overline{T}_2 = T_2/T_{\infty}$, $\overline{F} = (E_{\downarrow} + E_{\uparrow})/(2\sigma T_{\infty}^4)$, and $\overline{x} = x/x_0$.

Figure 1 presents the kinetics of decrease of the temperature and the density of radiation in the layer at four depths of it for a thickness of $x_0 = 0.2$ m. Since the problem is symmetric about the center of the layer, we studied depths $\overline{x} \le 0.5$. We consider two cases where the initial parameters are such that the process of cooling is very fast and slow.

According to the initial conditions, $\overline{T}_1 = \overline{T}_2 = 3.33$ at $t = 0$. As is seen from Fig. 1a, at the initial stage in the absence of heat transfer from the layer to the environment by heat conduction ($\alpha_1 = 0$) all portions of the medium have close temperatures. Cooling occurs only due to emission. Clearly, the surface layers cool faster than deeper layers. According to the Kirchhoff law, the particles, which have an absorptivity higher than the binding medium (100% of the energy incident on a particle changes to heat), emit more energy (on a constant-volume basis). Therefore, at any instant of time their temperature at the surface is lower than that of the binding medium. However, this difference disappears with increase in the depth. In this case, a strong effect of a large coefficient of heat exchange α_2 between the particles and the binding medium manifests itself and the temperatures of both components are equalized. The varying slope of the curves at different t in Fig. 1a and also in Fig. 1b indicates that with time the leading role in the kinetics is played by

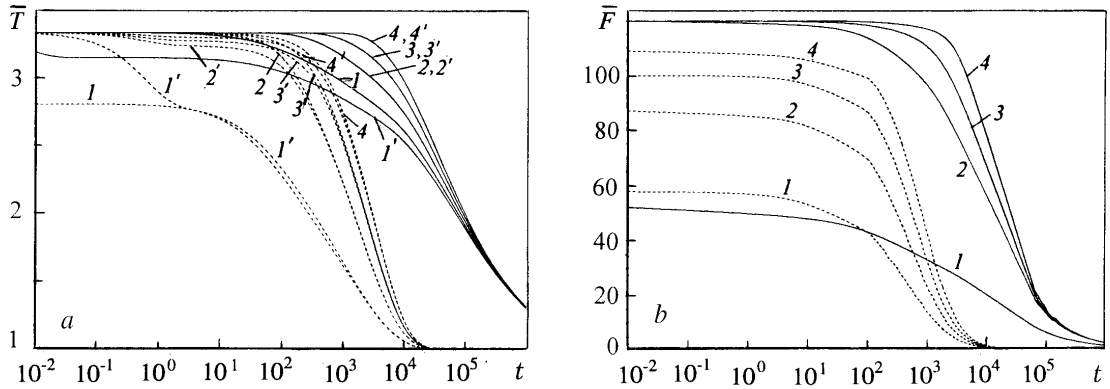
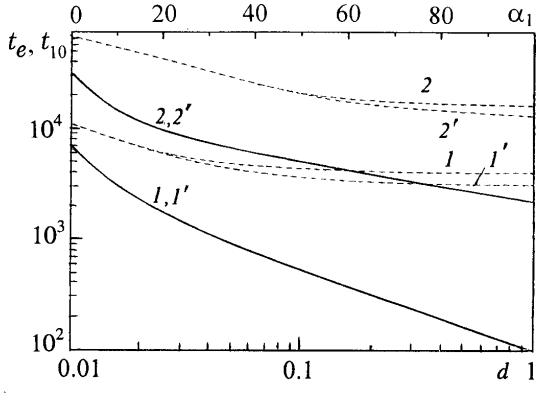


Fig. 1. Dependence of $\overline{T}$ (a) and $\overline{F}$ (b) on t at different depths of the layer. Solid curves: $\alpha_1 = 0$, $\alpha_2 = 8000$ m^{-1}, $\nu_2 = 0.01$, $d = 10^{-5}$ m, and $\eta_1 = 0.1$ W·m^{-1}·K^{-1}; dashed curves: $\alpha_1 = 50$ m^{-1}, $\alpha_2 = 800$ m^{-1}, $\nu_2 = 0.001$, $d = 10^{-3}$ m, and $\eta_1 = 1$ W·m^{-1}·K^{-1}. Unprimed figures — the binding medium; primed figures — particles; 1, 1′) $\overline{x} = 0$; 2, 2′) 0.125; 3, 3′) 0.25; 4, 4′) = 0.5. t, sec.

different parameters of the medium. In the given scale, a sharp dip of the curves is observed within about an hour after the onset of cooling.

The process is much faster if there is heat-conduction exchange of energy between the binding medium and the environment ($\alpha_1 = 50$ m^{-1}). Whereas in the first abstract case (absence of convection and $\alpha_1 = 0$), it takes tens of days for the temperatures of the layer and the environment to be equalized completely, in the second case this occurs in about one day. In this case, at first moments the particles have a much higher temperature than the medium, since it is precisely the latter that has a large coefficient α_1 (particles in low concentration do not contact the environment). But after an interval of about several seconds the values of $\overline{T}_1$ and $\overline{T}_2$ are equalized, in practice. This is caused by the manifestation of the mechanism of conductive (due to a large value of α_2) heat transfer from hotter particles to the binding medium. As the depth of the layer is approached, the temperature increases. The effect of α_2 on the differences between T_1 and T_2 disappears. A certain excess of T_1 over T_2 at the initial stage of cooling is explained by the higher emissive power of the particles.

The character of the light-field structure at different instants of time at different depths in the layer is seen in Fig. 1b. Clearly, $\overline{F} > 1$ as long as $\overline{T} > 1$, because only the excess of T over T_∞ forms an excess of the radiation density over the background density. Since at $\alpha_1 = 0$ the temperature in the layer is higher than at $\alpha_1 = 50$ m^{-1}, in the first case at most depths $\overline{F}$ is larger than in the second. However, at the surface itself the picture seems strange at first sight. At the initial stage of cooling, the dashed curve 1 lies higher than the solid curve 1, although the temperature of the surface at $\alpha_1 = 50$ m^{-1} is lower than at $\alpha_1 = 0$. This is explained by the fact that the light field at the site under consideration is determined not only by the temperature in it but also by radiation coming from other portions of the layer. This "make-up" depends on the transmission capacity of the medium. According to the scattering theory [6, 7], the latter is determined by the probability of survival of a photon $\Lambda = s_2/(\varepsilon_2 + k_1)$ and the optical thickness $\tau_0 = (\varepsilon_2 + k_1)x_0$ of the layer. The lower Λ and the larger τ_0, the less light is transmitted by the layer, and, consequently, by its individual portions. In our situation, two types of layers have the same Λ, which is equal to 0.5, and the optical thickness is 620 in first case and 2.6 in the second case. In the first case, make-up of the surface by light from deep strongly heated portions is virtually impossible due to the large τ_0, while in the second case it is very high, which explains the observed surface effect.



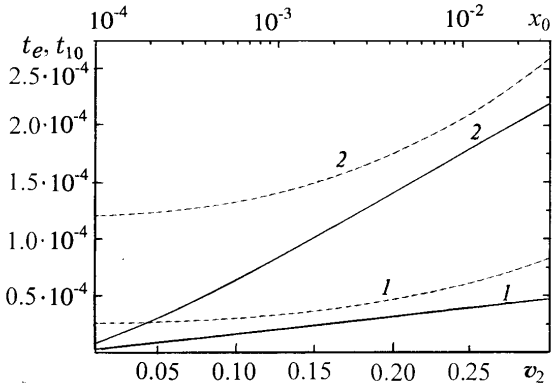


Fig. 2. Dependence of t_e and t_{10} on α_1 (solid curves) and d (dashed curves). Unprimed figures — the binding medium; primed figures — particles; 1, 1′) t_e; 2, 2′) t_{10}. t, sec; α, m^{-1}; d, mm.

Fig. 3. Dependence of t_e and t_{10} on x_0 (solid curves) and v_2 (dashed curves). 1) t_e; 2) t_{10}. t, sec; x_0, mm.

Above we have considered the processes of cooling of just two layers with distinctly different optical and thermophysical characteristics. This does not offer a complete idea of the effect of each parameter on the kinetics of cooling. We introduce the concept of the rate of cooling of a layer which is characterized by the reciprocal of the time t_n during which the initial excess of the temperature of the layer surface over the temperature of the environment $T_{in} - T_\infty$ decreases n-fold. Let n be equal to e = 2.71 and 10. The process of cooling is multiparametric, i.e., at different stages it is determined by different time characteristics (we will be dealing with them in what follows) and the dependences of t_n on these parameters may be dissimilar at different n. That is why we have taken two values of n. We analyze the effect of α_1, d, v_2, and x_0 on the rate of cooling of the layer. In this case, we will speak of the rates of cooling of the binding medium and the particles on the layer surface, which may not coincide.

Figure 2 shows the dependences of t_e and t_{10} on α_1 and d. When the effect of α_1 is considered, $v_2 = 10^{-3}$, $d = 10^{-4}$ m, $\eta_1 = 1$ W·m^{-1}·K^{-1}, $\alpha_2 = 8 \cdot 10^4$ m^{-1}, and $x_0 = 0.2$ m, and when the effect of d is considered, $v_2 = 10^{-3}$, $\alpha_1 = 10$ m^{-1}, $\eta_1 = 0.05$ W·m^{-1}·K^{-1}, $\eta_2 = 20$ W·m^{-1}·K^{-1}, and $x_0 = 0.2$ m. It should be noted that as d changes, α_2 also changes. It is rather difficult to find a correlation between them. Proceeding from a number of physical concepts and certain experimental results, in this calculation we used the relation $\alpha_2 = 8\eta_1/\eta_2 d$, where η_2 is the coefficient of thermal conductivity of a particle. It must be noted that η is of the order of 0.1 W·m^{-1}·K^{-1} for dielectric materials, 1 — for semiconductors, and 10 — for metals. A certain groundlessness of the above relation is justified by the fact that a relatively wide variation of α_2 does not drastically change the values of t_e and t_{10}. It is seen from the figure that as the coefficient of heat exchange between the binding medium and the environment increases the values of t_e and t_{10} clearly decrease. In this case, T_1 and T_2 are equal in value; therefore, the medium cools as a united whole at different α_1. The quantity t_e within the considered range of α_1 changes 65-fold and t_{10} changes 15-fold.

We consider t_e and t_{10} as functions of d. They are also decreasing, but, in contrast to the previous case, the curves for the binding medium and the particles somewhat differ. The quantity t_e within the considered range of d changes about 6.3-fold and that of t_{10} changes 3-fold. The faster cooling with increase in d is explained as follows. The increase in d at a constant v_2 results in a decrease in the optical thickness, which allows the radiation and, consequently, the heat to come easily from the depth to the surface and be emitted from it. As a result, the thermal energy is exhausted faster in the layer.

Figure 3 presents the dependences of t_e and t_{10} on v_2 and x_0. When the effect of v_2 is considered, $\alpha_1 = 10$ m^{-1}, $d = 10^{-4}$ m, $\eta_1 = 1$ W·m^{-1}·K^{-1}, $\alpha_2 = 8 \cdot 10^4$ m^{-1}, and $x_0 = 0.2$ m, and when the effect of x_0 is

TABLE 1. Values of τ_{x_0}, τ_{12}, τ_c (sec), and A for Different Parameters of the Medium

d, m	$v_2 = 0.001$			$v_2 = 0.01$		
	x_0, m					
	0.01	0.03	0.1	0.01	0.03	0.1
1–5	1 + 2 8.3 – 3 3.6 + 3 0.92	9 + 2 8.3 – 3 1 + 4 0.95	1 + 4 8.3 – 3 – 0.95	1 + 2 8.3 – 4 3.6 + 3 0.95	9 + 2 8.3 – 4 1 + 4 0.95	1 + 4 8.3 – 4 – 0.95
1–4	1 + 2 8.3 – 1 1.2 + 4 0.28	9 + 2 8.3 – 1 1.6 + 4 0.61	1 + 4 8.3 – 1 – 0.92	1 + 2 8.3 – 2 3.6 + 3 0.92	9 + 2 8.3 – 2 1 + 4 0.95	1 + 4 8.3 – 2 – 0.95
1–3	1 + 2 8.3 + 1 1 + 5 0.032	9 + 2 8.3 + 1 1.1 + 5 0.09	1 + 4 8.3 + 1 – 0.28	1 + 2 8.3 + 0 1.2 + 4 0.28	9 + 2 8.3 + 0 1.6 + 4 0.61	1 + 4 8.3 + 0 – 0.92

considered, $\alpha_1 = 10$ m^{-1}, $d = 10^{-4}$ m, $\eta_1 = 1$ W·m^{-1}·K^{-1}, $\alpha_2 = 8 \cdot 10^4$ m^{-1}, and $v_2 = 10^{-3}$. It is seen from the figure that the values of t_e and t_{10} increase with v_2. The explanation for this fact is the same as with changes in d (Fig. 2). As v_2 increases, τ_0 also increases, which restrains the transfer of heat in the form of a radiation flow from the depth of the medium and retards the cooling of the layer, i.e., causes t_e and t_{10} to increase. The quantity t_e within the considered range of v_2 changes 3.4-fold, and t_{10} changes 2.17-fold. The values of t_e and t_{10} for the binding medium and the particles coincide.

As the thickness of the layer increases, the layer cools more slowly due to a larger amount of accumulated energy. The temperatures of the particles and the binding medium remain the same due to the large value of α_2. Therefore, they cool according to the same law. Within the considered range of x_0, t_e changes 18-fold and t_{10} changes 29-fold.

Many of the features of the kinetics of cooling of a layer and the role of thermophysical, optical, and geometric parameters in this process can be revealed without solving the differential equations (8)–(10). We transform Eqs. (10) and (11) to the form

$$\frac{\partial \overline{T}_1}{\partial t} = \frac{\overline{F} - \overline{T}_1^{\,4}}{\tau_{1E}} + \frac{1}{\tau_{x_0}} \frac{\partial^2 \overline{T}_1}{\partial \overline{x}^{\,2}} + \frac{\overline{T}_2 - \overline{T}_1}{\tau_{12}}, \quad \frac{\partial \overline{T}_2}{\partial t} = \frac{\overline{F} - \overline{T}_2^{\,4}}{\tau_{2E}} - \frac{\overline{T}_2 - \overline{T}_1}{\tau_{12} v_2}, \tag{12}$$

where τ_{1E}, τ_{2E}, τ_{x_0}, and τ_{12} are the times which have an obvious physical meaning: $\tau_{1E} = c\rho/(2k_1\sigma T_\infty^3)$ is the characteristic time of equalization of the temperature of the binding medium and the field of radiation in the layer, $\tau_{2E} = c\rho d/(3\sigma T_\infty^3)$ is the characteristic time of equalization of the temperature of the particles and the field of radiation, $\tau_{x_0} = c\rho x_0/\eta_1$ is the characteristic time of equalization of the temperature over the binding medium in the layer, and $\tau_{12} = c\rho d/(6v_2 h_2)$ is the characteristic time of temperature exchange between the particle and the binding medium.

Table 1 gives the values of the most important parameters for different characteristics of the layer: τ_{x_0} (the first number in the square) and τ_{12} (the second number in the square). The data are obtained at $\alpha_1 = 0$ and $\eta_1 = \eta_2 = 1$ W·m^{-1}·K^{-1}. From these data we can determine the order of times in which the layer manages to become uniformly heated when the temperatures of the binding medium and the particles are equalized if the equilibrium of the system is disturbed.

If $\alpha_1 = 0$ and τ_{x_0} and τ_{12} are much shorter than the entire process of cooling, we can obtain an analytical expression for the kinetics of cooling. In this case, heat balance for the layer is described by the equation

$$c\rho x_0 \frac{dT}{dt} = -2\sigma A\,(T^4 - T_\infty^4)\,, \tag{13}$$

where A is the absorption coefficient of the layer, which is equal to the ratio of the power of the absorbed light to the power of the light diffusely incident on the layer.

If we introduce the dimensionless temperature $\overline{T}$, then, instead of (13), we have

$$\frac{d\overline{T}}{dt} = -\frac{\overline{T}^4 - 1}{\tau_c}\,. \tag{14}$$

Here $\tau_c = c\rho x_0/2\sigma T_\infty^3 A$ is the characteristic time of cooling (an order of magnitude) of a uniformly heated layer when the process occurs only due to radiation from the surface. Its values are given in Table 1 (the third number in the square). The values of τ_c are not given for those cases where the conditions of uniform heating are violated.

The solution of (14) has the form

$$\frac{1}{4}\ln\frac{(T_\infty + T)\,(T_\infty - T_{in})}{(T_\infty - T)\,(T_\infty + T_{in})} + \frac{1}{2}\left(\arctan\frac{T}{T_\infty} - \arctan\frac{T_{in}}{T_\infty}\right) = \frac{t}{\tau_c}\,, \tag{15}$$

which for $T >> T_\infty$ is

$$T = \frac{T_{in}}{\left(1 + \dfrac{6\sigma T_{in}^3 A}{c\rho x_0}\,t\right)^{1/3}}\,. \tag{16}$$

Thus, we can determine A from the character of cooling T.

The absorption coefficient is $A = 1 - r(\tau_0) - t(\tau_0)$, where, according to [10], in strong absorption the reflection coefficient $r(\tau_0)$ of the layer with a thickness τ_0 is expressed in terms of the reflection coefficient r_∞ of an infinitely thick layer and the transmission coefficient $t(\tau_0)$ is expressed as follows: $r(\tau_0) = r_\infty[1 - t^2(\tau_0)]$. Here $r_\infty = \varphi s_2/(k_1 + k_2)$ and $t(\tau_0) = \exp\{1.1[1 - (1 - \varphi)\Lambda]\tau_0\}$. We set $k_1 = 0$ and $\varphi = 0.05$. Then $\Lambda = 0.5$, $r_\infty = 0.05$, $t(\tau_0) = \exp(-3.465 v_2 x_0/d)$, and

$$A = 1 - 0.05\,[1 - \exp(-6.93 v_2 x_0/d)] - \exp(-3.465 v_2 x_0/d)\,. \tag{17}$$

Table 1 also gives the values of A (the fourth number in the square).

Knowing the layer thickness, under certain conditions we can find the ratio v_2/d from the character of cooling.

For the considered case of absence of the dependence of the temperature on the coordinate, it has been shown in [11] that the radiation density is

$$E(\tau) = E_\downarrow(\tau) + E_\downarrow(\tau) = \sigma T^4 \{2 - 2r(\tau)\,r(\tau_0 - \tau) - t(\tau)\,[1 + r(\tau_0 - \tau)] - \tag{18}$$

$$- t(\tau_0 - \tau)\,[1 + r(\tau)]\}/[1 - r(\tau)\,r(\tau_0 - \tau)]\,.$$

It follows from (18) that the distribution $E(\tau)$ is symmetric about the plane $\tau = \tau_0/2$. At τ equal to 0 or τ_0 we have

$$E(0) = E(\tau_0) = \sigma T^4 [1 - r(\tau_0) - t(\tau_0)], \tag{19}$$

and at $\tau = \tau_0/2$ we have

$$E(\tau_0/2) = 2\sigma T^4 \left\{1 - r^2(\tau_0/2) - t(\tau_0/2)[1 + r(\tau_0/2)]\right\}/[1 - r^2(\tau_0/2)]. \tag{20}$$

Having divided (20) by (19), at large τ_0 we obtain

$$E(\tau_0/2)/E(0) = 2/(1 - r_\infty), \tag{21}$$

and at small τ_0 we obtain

$$E(\tau_0/2)/E(0) = 1. \tag{22}$$

Relations (21) and (22) allow one to easily evaluate the difference between the illuminances at the center of the layer and at its edges for different optical parameters of the medium.

Thus, we have formulated differential equations for the flows of radiation and heat of a cooling, two-component, light-diffusing layer. Their solution is based on a specified finite-difference approximation of derivatives with respect to space, which reduces the problem to solution of the system of ordinary differential equations with respect to time for the temperature and the radiation flows at each point. Changes in the temperature and the radiation field over the layer thickness have been analyzed for different optical and thermophysical parameters of the medium. The interpretation of the observed phenomena has been given. The concepts of a number of characteristic times have been presented; by calculating these times one can evaluate the process of cooling without resorting to solution of differential equations. For the layer where the temperature is independent of the coordinate, we have obtained simple analytical expressions for the temperature and the density of the light field. It is shown that under certain conditions the ratio of the volume concentration of the particles to their diameter can be calculated from the rate of cooling.

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